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Instabilities in Strong Magnetic Fields in String Theory*

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ABSTRACT

We construct groundstates of the string with non-zero mass gap and non-trivial chromo-magnetic fields as well as curvature. The exact spectrum as function of the chromo-magnetic fields and curvature is derived. We examine the behavior of the spectrum, and find that there is a maximal value for the magnetic field $H_{\rm max} \sim M^2_{\rm Plank}$. At this value all states that couple to the magnetic field become infinitely massive and decouple. We also find tachyonic instabilities for strong background fields of the order $\mathcal{O}(\mu M_{\rm Plank})$ where μ is the mass gap of the theory. Unlike the field theory case, we find that such ground states become stable again for magnetic fields of the order $\mathcal{O}(M^2_{\rm Plank})$. The implications of these results are discussed.

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1 Introduction

In four-dimensional Heterotic or type II Superstrings it is possible, in principle, to understand the response of the theory to non-zero gauge or gravitational field backgrounds including quantum corrections. This problem is difficult in its full generality since we are working in a first quantized framework. In certain special cases, however, there is an underlying 2-d superconformal theory which is well understood and which describes exactly (via marginal deformations) the turning-on of non-trivial gauge and gravitational backgrounds. This exact description goes beyond the linearized approximation. In such cases, the spectrum can be calculated exactly, and it can provide interesting clues about the physics of the theory.

In field theory (excluding gravity) the energy shifts of a state due to the magnetic field have been investigated long ago [1, 2, 3]. The classical field theory formula for the energy of a state with spin S, mass M and charge e in a magnetic field H pointing in the third direction is:

$$E^2 = p_3^2 + M^2 + |eH|(2n + 1 - gS) \tag{1.1}$$

were $g = 1/S$ for minimally coupled states and $n = 1, 2, ...$ labels the Landau levels. It is obvious from (1.1) that minimally coupled particles cannot become tachyonic, so the theory is stable. For non-minimally coupled particles, however, the factor $2n + 1 - gS$ can become negative and instabilities thus appear. For example, in non-abelian gauge theories, there are particles which are not minimally coupled. In the standard model, the $W^\pm$ bosons have $g = 2$ and $S = \pm 1$. From (1.1) we obtain that the spontaneously broken phase in the standard model is thus unstable for magnetic fields that satisfy [2, 3]

$$|H| \geq \frac{M_W^2}{|e|} \tag{1.2}$$

A phase transition has to occur by a condensation of W bosons, most probably to a phase where the magnetic field is confined (localized) in a tube, [3]. This behavior should be contrasted to the constant electric field case where there is particle production [4] for any non-zero value of the electric field, but the vacuum is stable (although the particle emission tends to decrease the electric field).

The instabilities present for constant magnetic fields are still present in general for slowly varying (long range) magnetic fields. For a non-abelian gauge theory in the unbroken phase, since the mass gap is classically zero, we deduce from (1.1) that the trivial vacuum ($A_\mu^a = 0$) is unstable even for infinitesimally small chromo-magnetic fields. This provides already an indication at the classical level that the trivial vacuum is not the correct vacuum in an unbroken non-abelian gauge theory. We know however, that such a theory acquires a non-perturbative mass gap, $\Lambda^2 \sim \mu^2 \exp[-16\pi^2 b_0/g^2]$ where g is the non-abelian gauge coupling. If in such a theory one managed to create a chromo-magnetic field then there would again appear an instability and the theory would again confine the field in a flux tube.

In string theory, non-minimal gauge couplings are present not only in the massless sector but also in the massive (stringy) sectors [5]. Thus one would expect similar instabilities. Since in string theory there are states with arbitrary large values of spin and one can naively expect that if g does not decrease fast enough with the spin (as is the case in open strings where $g = 2$ [5]) then for states with large spin an arbitrarily small magnetic field would destabilize the theory. This behavior would imply that the trivial vacuum is unstable. This does not happen however since the masses of particles with spin also become large when the spin gets large.

The spectrum of open bosonic strings in constant magnetic fields was derived in [6]. Open bosonic strings however, contain tachyons even in the absence of background fields. It is thus more interesting to investigate open superstrings which are tachyon-free. This was done in [7]. It was found that for weak magnetic fields the field theory formula (1.1) is obtained, and there are similar instabilities.

In closed superstring theory however, one is forced to include the effects of gravity. A constant magnetic field for example carries energy, thus, the space cannot remain flat anymore. The interesting question in this context is, to what extend, the gravitational backreaction changes the behavior seen in field theory and open string theory. As we will see the gravitational backreaction is important and gives rise to interesting new phenomena in strong magnetic fields.

Such questions can have potential interesting applications in string cosmology since long range magnetic fields can be produced at early stages in the history of the universe where field theoretic behavior can be quite different from the stringy one.

The first example of an exact electromagnetic solution to closed string theory was described in [8]. The solution included both an electric and magnetic field (corresponding to the electrovac solution of supergravity). In [9] another exact closed string solution was presented (among others) which corresponded to a Dirac monopole over S^2. More recently, several other magnetic solutions were presented corresponding to localized [10] or covariantly constant magnetic fields [11]. The spectrum of these magnetic solutions seems to have a different behavior as a function of the magnetic field, compared to the situation treated in this paper. The reason for this is that [11] considered magnetic solutions where the gravitational backreaction produces a non-static metric. "Internal" magnetic fields of the type described in [9] were also considered recently [12] in order to break spacetime supersymmetry.

Here we will study the effects of covariantly constant (chromo)magnetic fields, $H^a_i = \epsilon^{ijk}F^a_{jk}$ and constant curvature $\mathcal{R}^{il} = \epsilon^{ijk}\epsilon^{lmn}\mathcal{R}_{jm,kn}$, in four-dimensional closed superstrings. The relevant framework was developed in [13] where ground states were found, with a continuous (almost constant) magnetic field in a weakly curved space. We will describe here the relevant framework and physics of such backgrounds. More details and conventions can be found in [14].

In the heterotic string (where the left moving sector is N=1 supersymmetric) the part of the σ-model action which corresponds to a gauge field background $A^a_\mu(x)$ is

$$V = (A^a_\mu(x)\partial x^\mu + F^a_{ij}(x)\psi^i\psi^j)\bar{J}^a \tag{1.3}$$

where F^a_{ij} is the field strength of A^a_μ with tangent space indices, eg. $F^a_{ij} = e^\mu_i e^\nu_j F^a_{\mu\nu}$ with e^μ_i

being the inverse vielbein, and ψ^i are left-moving world-sheet fermions with a normalized kinetic term. $\bar{J}^a$ is a right moving affine current.

Consider a string ground state with a flat non-compact (euclidean) spacetime ($\mathbb{R}^4$). The simplest case to consider is that of a constant magnetic field, $H_i^a = \epsilon^{ijk} F_{jk}^a$. Then the relevant vertex operator (1.3) becomes

$$V_{flat} = F_{ij}^a(\frac{1}{2}x^i \partial x^j + \psi^i \psi^j)\bar{J}^a \tag{1.4}$$

This vertex operator however, cannot be used to turn on the magnetic field since it is not marginal (when F_{ij}^a is constant). In other words, a constant magnetic field in flat space does not satisfy the string equations of motion, in particular the ones associated with the gravitational sector.

A way to bypass this problem we need to switch on more background fields. In [13] we achieved this in two steps. First, we found an exact string ground state in which $\mathbb{R}^4$ is replaced by $\mathbb{R} \times S^3$. The $\mathbb{R}$ part corresponds to free boson with background charge $Q = 1/\sqrt{k+2}$ while the S^3 part corresponds to an $SU(2)_k$ WZW model. For any (positive integer) k, the combined central charge is equal to that of $\mathbb{R}^4$. For large k, this background has a linear dilaton in the x^0 direction as well as an $SO(3)$-symmetric antisymmetric tensor on S^3, while the metric is the standard round metric on S^3 with constant curvature. On this space, there is an exactly marginal vertex operator for a magnetic field which is

$$V_m = H(J^3 + \psi^1\psi^2)\bar{J}^a \tag{1.5}$$

Here, J^3 is the left-moving current of the $SU(2)_k$ WZW model. V_m contains the only linear combination of J^3 and $\psi^1\psi^2$ that does not break the N=1 local supersymmetry. The exact marginality of this vertex operator is obvious since it is a product of a left times a right abelian current. This operator is unique up to an $SU(2)_L$ rotation.

We can observe that this vertex operator provides a well defined analog of V_{flat} in eq. (1.3) by looking at the large k limit. We will write the SU(2) group element as $g = \exp[i\vec{\sigma}\cdot\vec{x}/2]$ in which case $J^i = kTr[\sigma^i g^{-1}\partial g] = ik(\partial x^i + \epsilon^{ijk}x_j\partial x_k + \mathcal{O}(|x|^3))$. In the flat limit the first term corresponds to a constant gauge field and thus pure gauge so the only relevant term is the second one which corresponds to constant magnetic field in flat space. The fact $\pi_2(S^3) = 0$ explains in a different way why there is no quantization condition on H.

There is another exactly marginal perturbation in the background above that turns on fields in the gravitational sector. The relevant perturbation is

$$V_{grav} = \mathcal{R}(J^3 + \psi^1\psi^2)\bar{J}^3 \tag{1.6}$$

This perturbation modifies the metric, antisymmetric tensor and dilaton [13]. For type II strings the relevant perturbation is

$$V_{grav}^{II} = \mathcal{R}(J^3 + \psi^1\psi^2)(\bar{J}^3 + \bar{\psi}^1\bar{\psi}^2) \tag{1.7}$$

We will not describe this perturbation further. They have been studied using the results of [15] and we refer the interested reader to [14] for more.

The space we are using, $\mathbb{R} \times S^3$ is such that the spectrum has a mass gap μ^2. In particular all gauge symmetries are broken spontaneously. This breaking however is not the standard breaking due to a constant expectation value of a scalar but due to non-trivial expectation values of the fields in the universal sector (graviton, antisymmetric tensor and dilaton).

2 Effective Field Theory Analysis

The starting 4-d spacetime (we will use Euclidean signature here) is described by the $SO(3)_{k/2} \times \mathbb{R}_Q$ CFT. The heterotic σ-model that describes this space is*

$$S_{4d} = \frac{k}{4}\mathbf{I}_{SO(3)}(\alpha,\beta,\gamma) + \frac{1}{2\pi}\int d^2z\left[\partial x^0\bar{\partial}x^0 + \psi^0\bar{\partial}\psi^0 + \sum_{a=1}^{3}\psi^a\bar{\partial}\psi^a\right] + \frac{Q}{4\pi}\int\sqrt{g}R^{(2)}x^0 \quad (2.1)$$

while the SU(2) action can be written in Euler angles as

$$\mathbf{I}_{SO(3)}(\alpha,\beta,\gamma) = \frac{1}{2\pi}\int d^2z\left[\partial\alpha\bar{\partial}\alpha + \partial\beta\bar{\partial}\beta + \partial\gamma\bar{\partial}\gamma + 2\cos\beta\partial\alpha\bar{\partial}\gamma\right] \quad (2.2)$$

with $\beta \in [0,\pi]$, $\alpha,\gamma \in [0,2\pi]$ and k is a positive even integer. In the type II case we have to add also the right moving fermions $\bar{\psi}^i$, $1 \le i \le 4$. The fermions are free (this is a property valid for all supersymmetric WZW models).

Comparing with the general (bosonic) σ-model

$$S = \frac{1}{2\pi}\int d^2z(G_{\mu\nu} + B_{\mu\nu})\partial x^\mu\bar{\partial}x^\nu + \frac{1}{4\pi}\int\sqrt{g}R^{(2)}\Phi(x) \quad (2.3)$$

we can identify the non-zero background fields as

$$G_{00} = 1 \quad , \quad G_{\alpha\alpha} = G_{\beta\beta} = G_{\gamma\gamma} = \frac{k}{4} \quad , \quad G_{\alpha\gamma} = \frac{k}{4}\cos\beta \quad , \quad B_{\alpha\gamma} = \frac{k}{4}\cos\beta \quad (2.4)$$

$$\Phi = Qx^0 = \frac{x^0}{\sqrt{k+2}} \quad (2.5)$$

where the relation between Q and k comes from the requirement that the (heterotic) central charge should be $(6,4)$, in which case we have (4,0) superconformal invariance, [16].

The perturbation that turns on a chromo-magnetic field in the $\mu = 3$ direction is proportional to $(J^3 + \psi^1\psi^2)\bar{J}$ where $\bar{J}$ is a right moving current belonging to the Cartan subalgebra of the heterotic gauge group. It is normalized so that $\langle\bar{J}(1)\bar{J}(0)\rangle = k_g/2$. Since

$$J^3 = k(\partial\gamma + \cos\beta\partial\alpha) \quad , \quad \bar{J}^3 = k(\bar{\partial}\alpha + \cos\beta\bar{\partial}\gamma) \quad (2.6)$$

this perturbation changes the σ-model action in the following way:

$$\delta S_{4d} = \frac{\sqrt{kk_g}H}{2\pi}\int d^2z(\partial\gamma + \cos\beta\partial\alpha)\bar{J} \quad (2.7)$$

* In most formulae we set $\alpha' = 1$ unless stated otherwise.

In the type II case $\bar{J}$ is a bosonic current (it has a left moving partner) and we can easily show that the σ-model with action $S_{4d} + \delta S_{4d}$ is conformally invariant to all orders in α'.

Reading the spacetime backgrounds from (2.1), (2.7) is not entirely trivial but straightforward. In type II case (which corresponds to standard Kalutza-Klein reduction) the correct metric has an $A_\mu A_\nu$ term subtracted [17]. In the heterotic case there is a similar subtraction but the reason is different. It has to do with the anomaly in the holomorphic factorization of a boson.

The background fields have to be solutions (in leading order in α') to equations of motion stemming from the following spacetime action [18]:

$$S = \int d^4x \sqrt{G} e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - \frac{1}{12} H^2 - \frac{1}{4g^2} F^a_{\mu\nu} F^{a,\mu\nu} + \frac{\delta c}{3} \right] \tag{2.8}$$

where we have displayed a gauge field A^a_μ, (abelian or non-abelian) and set $g_{\text{string}} = 1$. The gauge coupling is $g^2 = 2/k_g$ due to the normalization of the affine currents,

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2g^2} \left[A^a_\mu F^a_{\nu\rho} - \frac{1}{3} f^{abc} A^a_\mu A^b_\nu A^c_\rho \right] + \text{cyclic permutations} \tag{2.9}$$

and f^{abc} are the structure constants of the gauge group. In this paper we will restrict ourselves to abelian gauge fields (in the cartan of a non-abelian gauge group).

It is not difficult now to read from (2.1), (2.7) the background fields that satisfy the effective action equations. The non-zero components are:

$$G_{00} = 1 \ , \ G_{\beta\beta} = \frac{k}{4} \ , \ G_{\alpha\gamma} = \frac{k}{4}(1 - 2H^2)\cos\beta \ , \ G_{\alpha\alpha} = \frac{k}{4}(1 - 2H^2\cos^2\beta) \tag{2.10}$$

$$G_{\gamma\gamma} = \frac{k}{4}(1 - 2H^2) \ , \ B_{\alpha\gamma} = \frac{k}{4}\cos\beta \ , \ A_a = g\sqrt{k} H \cos\beta \ , \ A_\gamma = g\sqrt{k} H \tag{2.11}$$

and the same dilaton as in (2.5). This background is exact to all orders in the α' expansion with the simple modification $k \to k + 2$.

It is interesting to note that

$$\sqrt{\det G} = \sqrt{1 - 2H^2} \left(\frac{k}{4} \right)^{3/2} \sin\beta \tag{2.12}$$

which indicates, as advertised earlier, that something special happens at $H_{max} = 1/\sqrt{2}$. At this point the curvature is regular. In fact, this is a boundary point where the states that couple to the magnetic field (i.e. states with non-zero left helicity and angular momentum and/or e) become infinitely massive and decouple. This is the same phenomenon as the degeneration of the Khäler structure on a two-torus ($\text{Im}U \to \infty$). Thus, this point is at the boundary of the magnetic field moduli space. This is very interesting since it implies the existence of a maximal magnetic field

$$|H| \leq H_{max} = \frac{1}{\sqrt{2}} \quad \text{or} \quad H_{\text{max}} = \frac{M^2_{\text{Plank}}}{\sqrt{2}} \tag{2.13}$$

in physical units with $M^2_{\rm Plank} = 1/\alpha' g^2_{\rm string}$ and where $g_{\rm string}$ is the string coupling constant.

We should note here that the deformation of the spherical geometry by the magnetic field is smooth for all ranges of parameters, even at the boundary point $H = 1/\sqrt{2}$. To monitor better the back-reaction of the effective field theory geometry we should first write the three-sphere with the round metric (2.4), as the (Hopf) fibration with S^1 as fiber and a two-sphere as base space:

$$ds^2_{3-{\rm sphere}} = \frac{k}{4}\left[ds^2_{2-{\rm sphere}} + (d\gamma + \cos\beta d\alpha)^2\right] \quad , \quad ds^2_{2-{\rm sphere}} = d\beta^2 + \sin^2\beta d\alpha^2 \tag{2.14}$$

The second term in (2.14) is the metric of the S^1 fiber, and its non-trivial dependence on α, β signals the non-triviality of the Hopf fibration. This metric has $SO(3) \times SO(3)$ symmetry.

The metric (2.10), (2.11) containing the backreaction to the non-zero magnetic field can be written as

$$ds^2 = \frac{k}{4}\left[ds^2_{2-{\rm sphere}} + (1 - 2H^2)(d\gamma + \cos\beta d\alpha)^2\right] \tag{2.15}$$

It is obvious from (2.15) that the magnetic field changes the radius of the fiber and breaks the $SO(3) \times SO(3)$ symmetry to the diagonal $SO(3)$. It is also obvious that at $H = 1/\sqrt{2}$, the radius of the fiber becomes zero. All the curvature invariants are smooth (and constant due to the $SO(3)$ symmetry)

3 Exact Spectrum and Instabilities

The exact spectrum of string theory in the magnetic background described in the last section can be computed by solving the associated conformal field theory, [14]. If we call M_L^2 the eigenvalues of L_0 and M_R^2 the eigenvalues of $\bar{L}_0$ we find

$$M_L^2 = -\frac{1}{2} + \frac{\mathcal{Q}^2}{2} + \sum_{i=1}^{3}\frac{\mathcal{Q}_i^2}{2} + \frac{(j+1/2)^2 - (\mathcal{Q}+I)^2}{k+2} + E_0 + \frac{\left[\frac{(\mathcal{Q}+I)}{\sqrt{k+2}} + eH\right]^2}{1-2H^2} \tag{3.16}$$

$$M_R^2 = -1 + \frac{\bar{\mathcal{P}}^2}{k_g} + \frac{(j+1/2)^2 - (\mathcal{Q}+I)^2}{k+2} + \bar{E}_0 + \frac{\left[\frac{(\mathcal{Q}+I)}{\sqrt{k+2}} + eH\right]^2}{1-2H^2} \tag{3.17}$$

where, the $-1/2$ is the universal intercept in the N=1 side, $\mathcal{Q}$ is the spacetime helicity, $\mathcal{Q}_i$ are the internal helicity operators (associated to the internal left-moving fermions), $E_0, \bar{E}_0$ contain the oscillator contributions as well as the internal lattice (or twisted) contributions, and $j = 0, 1, 2, \cdots, k/2$*, $j \geq |I| \in Z$. $\bar{\mathcal{P}}$ is the zero mode of the affine current associated to the relevant gauge group and $e = \sqrt{2}\bar{\mathcal{P}}/\sqrt{k_g}$. There is also the usual GSO projection $\mathcal{Q} + \sum_{i=1}^{3}\mathcal{Q}_i =$ odd integer.

We can see here another reason for the need of the SO(3) projection. We do not want half integral values of I to change the half-integrality of the spacetime helicity $\mathcal{Q}$. Since

*Remember that k is an even integer for $SO(3)$.

for physical states $M_L^2 = M_R^2$ it is enough to look at M_L^2 which in our conventions is the side that has $N = 1$ superconformal symmetry.

The first observation we can make here is to confirm the existence of a maximal magnetic field (2.13) suggested from the effective field theory analysis. It is obvious from (3.16,3.17) that at $H = 1/\sqrt{2}$ all states that couple to the magnetic field become infinitely massive.

It is not difficult to check that spacetime fermions have always positive mass square, a property required by unitarity.

For bosons though, states with non-zero helicity can become tachyonic for some range of values of the magnetic field. It can be shown that only helicity-one ($\mathcal{Q} = 1$) states can become tachyonic. Such states have also $E_0 = \mathcal{Q}_i = 0$ and $j = |I|$. Thus there are instabilities provided

$$\frac{1}{1-2H^2}\left(\frac{(1+I)}{\sqrt{k+2}} + eH\right)^2 + \frac{(|I|+1/2)^2 - (1+I)^2}{k+2} \leq 0 \tag{3.18}$$

and

$$\frac{1}{2(k+2)} \leq e^2 \leq 2 \tag{3.19}$$

Introducing the mass gap $\mu^2 = 1/(k+2)$ we obtain tachyonic instabilities when

$$H^{\rm crit}_{\rm min} \leq |H| \leq H^{\rm crit}_{\rm max} \tag{3.20}$$

with

$$H^{\rm crit}_{\rm min} = \frac{\mu}{|e|}\frac{1-\frac{\sqrt{3}}{2}\sqrt{1-\frac{1}{2}\left(\frac{\mu}{e}\right)^2}}{1+\frac{3}{2}\left(\frac{\mu}{e}\right)^2} \quad , \quad H^{\rm crit}_{\rm max} = \frac{\mu}{|e|}\frac{J+1+\sqrt{\left(J+\frac{3}{4}\right)\left(1-2\left(J+\frac{1}{2}\right)^2\frac{\mu^2}{e^2}\right)}}{1+\left(2J+\frac{3}{2}\right)\frac{\mu^2}{e^2}} \tag{3.21}$$

and

$$J = \text{integral} \quad \text{part} \quad \text{of} \quad -\frac{1}{2} + \frac{|e|}{\sqrt{2}\mu} \tag{3.22}$$

We note that for small μ and $|e| \sim \mathcal{O}(1)$ $H^{\rm crit}_{\rm min}$ is of order $\mathcal{O}(\mu)$. However $H^{\rm crit}_{\rm max}$ is below $H_{\rm max} = 1/\sqrt{2}$ by an amount of order $\mathcal{O}(\mu)$. Thus for small values of H there are no tachyons until a critical value $H^{\rm crit}_{\rm min}$ where the theory becomes unstable. For $|H| \geq H^{\rm crit}_{\rm max}$ the theory is stable again till the boundary $H = 1/\sqrt{2}$. It is interesting to note that if there is a charge in the theory with the value $|e| = \sqrt{2}\mu$ then $H^{\rm crit}_{\rm max} = 1/\sqrt{2}$ so there is no region of stability for large magnetic fields. For small μ there are always charges satisfying (3.19) which implies that there is always a magnetic instability. However even for $\mu = \mathcal{O}(1)$ the magnetic instability is present for standard gauge groups that have been considered in string model building (provided they have charged states in the perturbative spectrum).

The behavior above should be compared to the field theory behavior (1.1). There we have an instability provided there is a particle with $gS \geq 1$. Then the theory is unstable for

$$|H| \geq \frac{M^2}{|e|(gS-1)} \tag{3.23}$$

where M is the mass of the particle (or the mass gap). However there is no restauration of stability for large values of H. This happens in string theory due to the backreaction of gravity. There is also another difference. In field theory $H_{crit} \sim \mu^2$ while in string theory $H_{crit} \sim \mu M_{\text{Plank}}$ where we denoted by μ the mass gap in both cases. This is due to the different ways of breaking the gauge symmetry.

A discussion on the flat space limit ($\mu \to 0$) of these solutions can be found in [14].

4 Conclusions and Further Comments

We have presented a class of magnetic backgrounds in closed superstrings and their associated instabilities. Our starting point are superstring ground states with an adjustable mass gap μ^2 [13]. In such ground states all gauge symmetries are spontaneously broken.

Exact magnetic and gravitational solutions can then be constructed in such ground states as exactly marginal perturbations of the appropriate CFTs. In the magnetic case, there is a monopole-like magnetic field on S^3. The gravitational backreaction squashes mildly the S^3 keeping however an $SO(3)$ symmetry. We have calculated the exact spectrum as a function of the magnetic field. The first interesting observation is that, unlike field theory, there is a maximum value for the magnetic field $\sim M^2_{\text{Plank}}$. At this value the part of the spectrum that couples to the magnetic field becomes infinitely massive.

We find magnetic instabilities in such a background. In particular, for $H \sim \mathcal{O}(\mu M_{\text{Plank}})$ there is a magnetic instability, driven by helicity-one states that become tachyonic. The critical magnetic field scales differently from the field theory result, due the different mechanism of gauge symmetry breaking.

We also find that, unlike field theory, the theory becomes stable again for strong magnetic fields of the order $\sim \mathcal{O}(M^2_{\text{Plank}})$.

Such instabilities could be relevant in cosmological situations, or in black hole evaporation. In the cosmological context, there maybe solutions where one has time varying long range magnetic fields. If the time variation is adiabatic, then there might be a condensation which would screen and localize the magnetic fields. Also, instabilities can be used as (on-shell) guides to find the correct vacuum of string theory. Our knowledge in that respect is limited since we do not have an exact description of all possible deformations of a ground state in string theory.

Another subject of interest, where instabilities could be relevant is Hawking radiation. It is known in field theory that Hawking radiation has many common features with production of Schwinger pairs in the presence of a long range electric field. In open string theory it was found, [19] that this rate becomes infinite for a $finite$ electric field , $E_{\text{crit}} \sim M^2_{\text{string}}$ (unlike the field theory case) and this behavior is due to α' corrections. Notice also that in the open string it is M_{string} and not M_{Plank} that is relevant due to the absence of gravity.

It would be interesting to see if this behavior persists in the presence of gravity (which is absent to leading order in open strings) by studying the effect in closed strings. In fact we expect that gravitational effects will be important for $E \sim M^2_{\rm planck}$. For small $g_{\rm string}$ however, we can have $M_{\rm string} << M_{\rm planck}$ so we expect a similar behavior as in the case of open strings. It is plausible that similar higher order corrections modify the Hawking rate in such a way that macroscopic black hole are unstable in string theory. Such a calculation seems difficult to perform with today's technology but seems crucial to the understanding of stringy black holes.

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